

# INTRODUCTION TO GAME THEORY

Heinrich H. Nax

Bary S. R. Pradel ski

&

hnax@ethz.ch

bpradelski@ethz.ch

*February 19, 2018: Lecture 1*



Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# A game



## Rules:

- ① **Players:** All of you:  
**<https://scienceexperiment.online/classroom/r/G0mwhB>**
- ② **Actions:** Choose a number between 0 and 100
- ③ **Outcome:** The player with the number closest to half the average of all submitted numbers wins.
- ④ **Payoffs:** He will receive half the average in CHF, which I will pay out right after the game.
- ⑤ In case of several winners, divide payment by number of winners and pay all winners.

# A game



# Structure of today's lecture

- Part 1: A “sort-of” introduction to the theory of games
- Part 2: Course admin:
  - Aims and requirements
  - Talk schedule

# Acknowledgments

- Bary Pradelski (ETHZ)
- Peyton Young (Oxford, LSE)
- Bernhard von Stengel (LSE)
- Françoise Forges (Paris Dauphine)
- Paul Duetting (LSE)
- Jeff Shamma (Georgia Tech, KAUST)
- Joergen Weibull (Stockholm, TSE)
- Andreas Diekmann (ETHZ)
- Dirk Helbing (ETHZ)

# Game theory

A tour of its people, applications and concepts

- ① von Neumann
- ② Nash
- ③ Aumann, Schelling, Selten, Shapley
- ④ Today



John von Neumann (1903-1957)

# What is game theory?

- A mathematical language to express models of, as Myerson says: “conflict and cooperation between intelligent rational decision-makers”
- In other words, *interactive decision theory* (Aumann)
- Dates back to von Neumann & Morgenstern (1944)
- Most important solution concept: the Nash (1950) equilibrium



# Games and Non-Games

What is a game? And what is not a game?

# Uses of game theory

- *Prescriptive* agenda versus *descriptive* agenda
- “Reverse game theory”/mechanism design:
  - “in a design problem, the goal function is the main given, while the mechanism is the unknown.” (Hurwicz)
- The mechanism designer is a game designer. He studies
  - What agents would do in various games
  - And what game leads to the outcomes that are most desirable

# Game theory revolutionized several disciplines

- Biology (evolution, conflict, etc.)
- Social sciences (economics, sociology, political science, etc.)
- Computer science (algorithms, control, etc.)
  
- game theory is now applied widely (e.g. regulation, online auctions, distributed control, medical research, etc.)

## Its impact in economics (evaluated by Nobel prizes)

- 1972: **Ken Arrow** – general equilibrium
- 1994: **John Nash, Reinhard Selten, John Harsanyi** – solution concepts
- 2005: **Tom Schelling** and **Robert Aumann** – evolutionary game theory and common knowledge
- 2007: **Leonid Hurwicz, Eric Maskin, Roger Myerson** – mechanism design
- 2009: **Lin Ostrom** – economic governance, the commons
- 2012: **Al Roth** and **Lloyd Shapley** – market design
- 2014: **Jean Tirole** – markets and regulation
- 2016: **Oliver Hart** and **Bengt Holmström** – contract theory
- 2017: **Richard Thaler** – limited rationality, social preferences

# Part 1: game theory

## “Introduction” / Tour of game theory

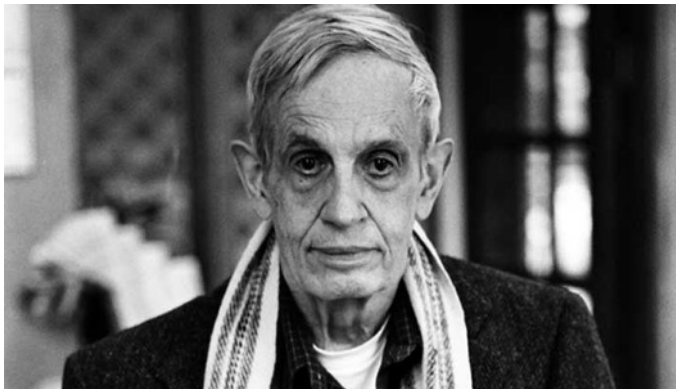
### Non-cooperative game theory

- No binding contracts can be written
- Players are individuals
- Main solution concepts:
  - Nash equilibrium
  - Strong equilibrium

### Cooperative game theory

- Binding contract can be written
- Players are individuals and coalitions of individuals
- Main solution concepts:
  - Core
  - Shapley value

# Noncooperative game theory



John Nash (1928-2015)

# A noncooperative game (normal-form)

- **players:**  $N = \{1, 2, \dots, n\}$  (finite)
- **actions/strategies:** (each player chooses  $s_i$  from his own finite strategy set;  $S_i$  for each  $i \in N$ )
  - resulting strategy combination:  $s = (s_1, \dots, s_n) \in (S_i)_{i \in N}$
- **payoffs:**  $u_i = u_i(s)$ 
  - payoffs resulting from the outcome of the game determined by  $s$

## Some 2-player examples

- **Prisoner's dilemma** – social dilemma, tragedy of the commons, free-riding
  - Conflict between individual and collective incentives
- **Harmony** – aligned incentives
  - No conflict between individual and collective incentives
- **Battle of the Sexes** – coordination
  - Conflict and alignment of individual and collective incentives
- **Hawk dove/Snowdrift** – anti-coordination
  - Conflict and alignment of individual and collective incentives
- **Matching pennies** – zero-sum, rock-paper-scissor
  - Conflict of individual incentives



		Player 2	
		Heads	Tails
Player 1	Heads	1,-1	-1,1
	Tails	-1,1	1,-1

## Matching pennies

		Confess A	Stay quiet A
Confess B		-6 -6	-10 0
Stay quiet B		0 -10	-2 -2

Prisoner's dilemma

		WOMAN	
		Boxing	Shopping
MAN	Boxing	2,1	0,0
	Shopping	0,0	1,2

Battle of the sexes

		Player 2	
		Hawk	Dove
Player 1	Hawk	-2,-2	4,0
	Dove	0,4	2,2

## Hawk-Dove game

		Company B	
		Cooperate	Not Cooperate
Company A	Cooperate	9,9	4,7
	Not Cooperate	7,4	3,3

Harmony game

# Equilibrium

## Equilibrium/solution concept:

An **equilibrium/solution** is a rule that maps the structure of a game into an equilibrium set of strategies  $s^*$ .

# Nash Equilibrium

## Definition: Best-response

Player  $i$ 's **best-response** (or, reply) to the strategies  $s_{-i}$  played by all others is the strategy  $s_i^* \in S_i$  such that

$$u_i(s_i^*, s_{-i}) \geq u_i(s'_i, s_{-i}) \quad \forall s'_i \in S_i \text{ and } s'_i \neq s_i^*$$

## Definition: (Pure-strategy) Nash equilibrium

All strategies are *mutual best responses*:

$$u_i(s_i^*, s_{-i}) \geq u_i(s'_i, s_{-i}) \quad \forall s'_i \in S_i \text{ and } s'_i \neq s_i^*$$

		Confess A	Stay quiet A
Confess B		-6	-10
Stay quiet B		0	-2

**Prisoner's dilemma:** both players confess (defect)



		WOMAN	
		Boxing	Shopping
MAN	Boxing	2,1	0,0
	Shopping	0,0	1,2

**Battle of the sexes:** coordinate on either option

		Player 2	
		Heads	Tails
Player 1	Heads	1,-1	-1,1
	Tails	-1,1	1,-1

Matching pennies: none (in pure strategies)

		Player 2	
		Hawk	Dove
Player 1	Hawk	-2,-2	4,0
	Dove	0,4	2,2

**Hawk-dove:** either of the two hawk-dove outcomes

		Company B	
		Cooperate	Not Cooperate
Company A	Cooperate	9,9	4,7
	Not Cooperate	7,4	3,3

Harmony: both cooperate

## Pure-strategy N.E. for our 2-player examples

- **Prisoner's dilemma** – social dilemma
  - Unique NE – socially undesirable outcome
- **Harmony** – aligned incentives
  - Unique NE – socially desirable outcome
- **Battle of the Sexes** – coordination
  - Two NE – both Pareto-optimal
- **Hawk dove/Snowdrift** – anti-coordination
  - Two NE – Pareto-optimal, but perhaps Dove-Dove “better”
- **Matching pennies** – zero-sum, rock-paper-scissor
  - No (pure-strategy) NE

# How about our initial game

Remember the rules were:

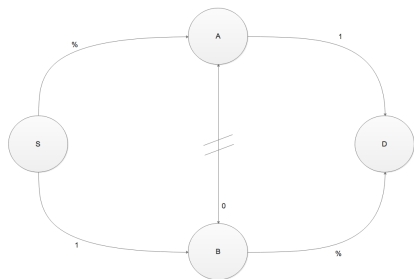
- ① Choose a number between 0 and 100
- ② The player with the number closest to half the average of all submitted numbers wins half the average in CHF

What is the Nash Equilibrium?

0

Actually, there are some Nash equilibria where players play 1 due to the fact that you earn 0 when you play 0... we will get back to this in detail later.

# Braess' Paradox



New road worsens congestion!

The story:

- 60 people travel from S to D
- Initially, there is no middle road. The NE is such that 30 people travel one way, the others the other way, and each driver drives 90 mins.
- A middle road is build. This road is super efficient. Now everyone will use it and drive the same route, and the NE will worsen to 119/120 mins.



# Cooperative games

The Nash equilibrium may not coincide with the outcome that is collectively preferable. Can players “cooperate” to achieve such an outcome?

- Suppose players can write **binding agreements** and directly **transfer utility**– e.g.:
    - *Contract 1*: Player 1 plays ‘Hawk’, player 2 plays ‘Dove’. Of the total payoffs, 1 and 2 receive equal shares.
- or
- *Contract 2*: Both players play ‘Boxing’. Of the total payoffs, Man gets 1.6 and Woman gets 1.4.

Then the **value** of the game in terms of a cooperative agreement is generally greater than the sum of the payoffs from the Nash equilibrium.

		Confess A	Stay quiet A
Confess B		-6	-10
Stay quiet B		0	-2

$$v(12) = -2 - 2 = -4$$

$$v(1) = v(2) = -6$$

Cooperative value  $= v(12) > v(1) + v(2) =$  Nash equilibrium payoffs

		WOMAN	
		Boxing	Shopping
MAN	Boxing	2,1	0,0
	Shopping	0,0	1,2

$$v(12) = 1 + 2 = 3$$

$$v(1) = v(2) = 0$$

Cooperative value=Nash equilibrium payoffs= $v(12) > v(1) + v(2)$ : payoffs can be split differently/more evenly

		Dawn	
		Hawk	Dove
Gary	Hawk	-2,-2	4,0
	Dove	0,4	2,2

$$v(12) = 4 + 0 = 2 + 2 = 4$$

$$v(1) = v(2) = -2$$

Cooperative value=Nash equilibrium payoffs= $v(12) > v(1) + v(2)$ : payoffs can be split differently/more evenly, achievable by dove-dove

		Company B	
		Cooperate	Not Cooperate
Company A	Cooperate	9,9	4,7
	Not Cooperate	7,4	3,3

$$v(12) = 9 + 9 = 18$$

$$v(1) = v(2) = 3$$

Cooperative value=Nash equilibrium payoffs= $v(12) > v(1) + v(2)$ , but payoffs can be split differently/more evenly

## Part 2: course admin

- Information about the course, and materials/slides of speakers, will be made available at

<http://www.coss.ethz.ch/education/GT.html>

<http://gametheory.online>

- Also, please contact us directly if you have any questions about the course:
  - Heinrich: [hnax@ethz.ch](mailto:hnax@ethz.ch)
  - Bary: [bpradelski@ethz.ch](mailto:bpradelski@ethz.ch)

# Drop-In Office Hours

Heinrich Nax

Monday

19.02.; 26.02.; 09.04; 23.04.; 30.04.

**15:00-16:00**

CLD C3

Bary Pradelski

Monday

05.03.; 12.03.; 19.03; 26.03.; 07.05.

**15:00-16:00**

CLD C5

# Schedule (preliminary) I

1) 19.02.	Introduction: a quick tour of game theory	Heinrich Nax
2) 26.02.	Cooperative game theory <ul style="list-style-type: none"> <li>●Core and Shapley value</li> <li>●Matching markets</li> </ul>	Heinrich Nax
3) 05.03.	Non-cooperative game theory: Normal form <ul style="list-style-type: none"> <li>●Utilities</li> <li>●Best replies</li> </ul>	Bary Pradelski
4) 12.03.	The Nash equilibrium <ul style="list-style-type: none"> <li>●Proof</li> <li>●Interpretations and refinements</li> </ul>	Bary Pradelski
5) 19.03.	Non-cooperative game theory: dynamics <ul style="list-style-type: none"> <li>●Sub-game perfection and Bayes-Nash equilibrium</li> <li>●Repeated games</li> </ul>	Bary Pradelski
	PROBLEM SET 1	
6) 26.03.	Game theory: evolution <ul style="list-style-type: none"> <li>●Evolutionary game theory</li> <li>●Algorithms in computer science (Price of anarchy)</li> </ul>	Bary Pradelski



## Schedule (preliminary) II

7) 09.04.	Experimental game theory <ul style="list-style-type: none"> <li>● Observing human behavior/experiments</li> <li>● Behavioral game theory</li> </ul>	Heinrich Nax
8) 23.04.	Applications <ul style="list-style-type: none"> <li>● Common pool resources</li> <li>● Distributed control</li> </ul>	Heinrich Nax
9) 30.04.	Bargaining <ul style="list-style-type: none"> <li>● Solution concepts</li> <li>● Nash program</li> </ul>	Heinrich Nax
10) 07.05.	Auctions <ul style="list-style-type: none"> <li>● English, Dutch, Sealed, Open</li> <li>● Equivalence and Real-world examples: 3G, Google, etc</li> </ul>	Bary Pradelski
11) 14.05.	EXAM	
12) 28.05.	Master class	t.b.c.

# Requirements

- ① Regularly attend and, please!, participate in seminar
- ② Problem set:
  - ① does not count toward final mark
- ③ **Exam: 14/5/2017**

# Finally, let's play again!



You remember the game:

- ① Choose a number between 0 and 100
  - Submit this number again:  
**<https://scienceexperiment.online/classroom/r/I9yY2o>**
- ② The player with the number that is closest to half the average of all submitted numbers wins half the average CHF (divided by number of winners)



## Nash equilibrium:

As a **non-cooperative game**, ...the Nash Equilibrium has total earnings of 0 (or 1).

# How about our initial game as a cooperative game

Cooperate or not?

- If all players submit 0, the average is 0: 0 earnings
- If all players submit 100, the average is 100: each player earns  $100/2n$
- Cooperatively, total earning could be 50!
- But what if all others submit 100, but one guy submits 50?
  - Then he wins and his earnings will be 50 instead of  $50/n$  ...

Cooperative values:

$$v(N) = 50$$

$$v(i) = 0$$

THANKS EVERYBODY

See you next week!

and keep checking the website for new materials as we progress:

<http://www.coss.ethz.ch/education/GT.html>

<http://gametheory.online>