Name: ______________________________

Student number: _ _ - _ _ _ - _ _ _

Exercises for "Introduction to Game Theory"

Heinrich H. Nax & Bary S. R. Pradelski

March 19, 2018
Due: March 26, 2018

1 Cooperative game theory

Exercise 1.1

Marginal contributions

1. If the value of coalition (A,B,C) is v(A,B,C)=100, and the value of coalition (A,B) is v(A,B)=30, and the value of C is v(C)=20, what is the marginal contribution of player C to coalition (A,B,C)?
2. If v(A)=20 and v(B)=0, what is the marginal contribution of B to (A,B)?
3. What is the marginal contribution of A to (A,B)?

Exercise 1.2

Superadditivity and the core

1. If the value of coalition N=(A,B,C) is v(N)=100, and the value of the coalition of pairs (i,j) is v(i,j)=30 for all pairs (i,j), and the value of each singleton i is v(i)=0, is the game superadditive?
2. Is the allocation (50,25,25) a core allocation?
3. Is the allocation (30,30,30) a core allocation?
4. Is the allocation (40,10,50) a core allocation?
5. Is the allocation (80,10,10) a core allocation?

Exercise 1.3

Cooperative games

A parliament is made up of four political parties, A, B, C, and D, which have 45, 25, 15, and 15 representatives, respectively. They are to vote on whether to pass a \$100 million spending bill and how much of this amount should be controlled by each of the parties. A majority vote, that is, a minimum of 51 votes, is required in order to pass any legislation, and if the bill does not pass then every party gets zero to spend.

1. Which coalitions of parties can obtain a majority?
2. Is the core non-empty? Can you find a core allocation (if so, write one down)?
3. Calculate the Shapley value.

Exercise 1.4

Cost sharing

Consider three neighboring municipalities, A, B, and C, who can supply themselves with municipal water either by building separate facilities or by building a joint water supply facility.

We suppose that the joint facility is cheaper to construct than the separate projects due to economies of scale. The quantity of water to be supplied to each municipality is assumed given. The problem is then how to divide the costs among them.

We can think of costs as "negative values". Suppose the costs of water supply translate into values such that v(A)=-6.5, v(B)=-4.2, v(C)=-1.5, v(A,B)=-10.3, v(B,C)=-5.3, v(A,C)=-8.0, v(A,B,C)=-10.6.

1. Is the game superadditive?
2. Is the core non-empty? Can you find a core allocation (if so, write one down)?
3. Calculate the Shapley value.

2 Preferences and utilities

Exercise 2.1

Suppose a decision maker is facing a choice over a finite set X and he has the binary preference relation $\succ$ over X.

1. Which of the following is true? (possibly several)
   (a) If $\succ$ is transitive there exists a utility function for $\succ$

(b) If $\succ$ is transitive, complete, and satisfies independence of irrelevant alternatives there exists a utility function for $\succ$

(c) If $\succ$ is transitive and complete there exists a utility function for $\succ$.

2. Give the definition of a utility function for a preference ordering $\succ$

Exercise 2.2

1. Define the Bernoulli function for preferences $\succ$ on X representing a decision maker's preferences over lotteries over a finite set T.
2. Further define the associated von Neumann-Morgenstern utility function.
3. When does a von Neumann-Morgenstern utility function exist for a preference relation $\succ$

Exercise 2.3

Suppose Tic, Tac, and Toe play the following simultaneous game. They each decide whether or not to go to the playground.

- if an odd number turns up at the playground Tic wins
- if exactly two people turn up at the playground Tac wins
- if nobody turns up at the playground Toe wins

Suppose that each player prefers winning to loosing and is indifferent between any two outcomes where he is loosing.

Formulate this game in a normal form with utility functions taking values 0 or 1.

3 Non-cooperative game theory

Exercise 3.1

Consider the two-player game with normal form:

	L	R
T	$7,6$	$0,5$
B	$2,0$	$4,3$

1. Find all Nash equilibria (in pure and mixed strategies)
2. Draw the best-reply graph
3. Find the expected payoff for row and column player in each of the equilibria

Exercise 3.2

Consider the two-player game with normal form:

	l	m	r
T	$14,7$	$2,7$	$2,0$
M	$14,1$	$10,5$	$0,2$
B	$0,1$	$4,0$	$12,0$

1. Find all Nash equilibria (in pure and mixed strategies)
2. Find all pure-strategy perfect equilibria
3. Find the set of iteratively strictly dominated pure strategies for each player
4. Delete all iteratively strictly dominated pure strategies and do tasks 1-3 for the new game

Exercise 3.3

Consider the following application. Bonnie and Clyde are to divide their latest robbery of 1 unit of gold. Each of them has (von Neumann-Morgenstern) utility $u(g) = \sqrt{(g)}$ from receiving $g \in [0,1]$ share of the gold.

1. Suppose each of them has to submit a suggestion for her share, say $b \in [0,1]$ for Bonnie and $c \in [0,1]$ for Clyde. The suggestions are written down independently. If the suggestions are compatible, that is, $b+c \leq 1$, then they each get their suggested share, otherwise both get nothing.
   (a) Define this as a game
   (b) Find the set of pure strategy Nash equilibria
2. Now suppose that Bonnie submits her suggestion before Clyde and suppose that Clyde hears about Bonnie's suggestion before she makes hers.
   (a) Define this as a game
   (b) Find the set of pure strategy subgame-perfect equilibria
   (c) Give an example of a Nash equilibrium that is not subgame-perfect
3. Suppose Bonnie and Clyde are able to write binding contracts. Find the Shapley value of the game

Exercise 3.4

Suppose a player has a strictly dominant strategy in a normal form game.

1. Is the following statement true "She is sure to get her best possible outcome in any Nash equilibrium of the game"?
2. Explain your answer and give an example of a game that illustrates your answer.

Exercise 3.5

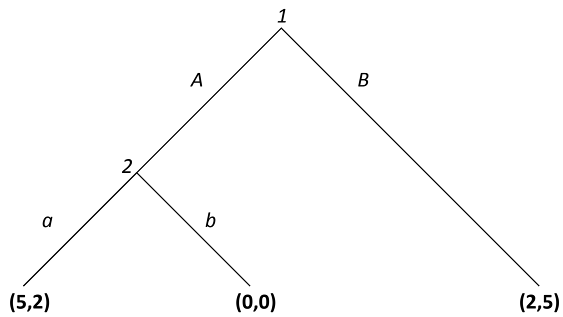
Consider the two-player game with normal form:

	L	R
T	$0, 0$	$-1, 1$
B	$X, -1$	$-2, -2$

1. Find all pure and mixed strategy Nash equilibria if $X > 0$
2. Find all pure and mixed strategy Nash equilibria if $X < 0$
3. If $X = 2$ is row player more likely to play T or column player more likely to play L? Which player gets the higher expected payoff.
4. Draw the best-reply graph for $X = 2, X = 0, X = -1$

Exercise 3.6

Given the following extensive form game:



1. Write down all subgames
2. Identify the pure strategy sets for both players
3. Write down the normal form representation of the game with player 1 as row player and player 2 as column player
4. Find the set of Nash equilibria of the game
5. Write down the definition of subgame perfect equilibrium
6. Find the unique subgame perfect equilibrium of the game