AUCTION THEORY

Heinrich H. Nax & Bary S. R. Pradelski

hnax@ethz.ch bpradelski@ethz.ch

May 15, 2017: Lecture 10

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Some logistics: Exam

- The exam will last 75 minutes, we will start at 17.15 – BE ON TIME
- NO helps allowed (no calculator, notes, etc.); ideally come with only some pens
- Please use provided answer templates

	Yes	No
Answer 1	X	
Answer 2		X
Answer 3		X

Answer 10	here you will write a free-text answer

Suppose you want to sell/buy one Microsoft stock, how would you do it?

Introduction

- Auctions are most widely-studied economic mechanism
- Auctions refer to arbitrary resource allocation problems with self-motivated participants: Auctioneer and bidders
- Auction (selling item(s)): one seller, multiple buyers

  e.g., selling a CD on eBay
- Reverse auction (buying item(s)): one buyer, multiple sellers

  e.g., procurement

  $\Rightarrow$ We will discuss auctions, but the same theory holds for reverse auctions

Historical note

- Reports that auctions were used in Babylon 500 B.C.
- 193 A.D. after having killed Emperor Pertinax, Praetorian guards sold the Roman Empire by means of an auction.

Where auctions are used nowadays

- Treasury auctions (bill, notes, Treasury bonds, securities)
- Transfer of assets from public to private sector
  - Right to drill oil, off-shore oil lease
  - Use the 4G spectrum
- Government and private corporations (construction, education, etc.)
- Private firms sell products (flowers, fish, tobacco, livestock, diamonds, . . .)
- Internet auctions
- Art auctions
- Procurement

Questions

- Seller has information problem: incomplete information about buyer's valuations (otherwise, he would just need to set price at maximum valuation of the buyer)
  - Which pricing scheme performs well in incomplete information settings?
  - Are auctions better suited for certain problems?
  - Does a specific type of auction yield greater revenue?

- Buyer: What are good bidding strategies?

How to compare auctions

Revenue

The revenue for the seller is the expected selling price.

Efficiency

The object ends up in the hands of the person, who values it the most (*resale does not increase efficiency*).

Open versus sealed bid auctions

Open bid auction

Bidders (competitors) are informed of each other and do also observe each others behavior.

Sealed bid auction (also closed bid auction)

Bidders are not informed of each other and do not observe each others behavior.

Private versus common value

Private value

The valuation of a bidder is independent of the valuations other bidders hold for the item. Further, no bidder knows with certainty the valuation of the other bidders.

(Pure) common value

The (pure) common value is the same for every bidder, but bidders have different private information about what that value actually is.

Example: In an auction of an oil field the amount of oil is unknown, but different bidders have different geological signals and learning another signal would change the valuation of a bidder.

Private or common value?

Correlated value

Correlated value

An agent's value of an item depends partly on its own preferences and partly on other's values for it.

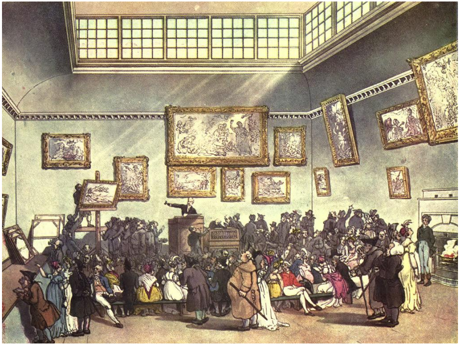
Example: THE DUTCH AUCTION

Open descending auction where the auctioneer calls out a rather high price, lowering it until a player indicates his interest. The first player doing so wins the object to the given price.

Dutch flower auctions

Example: THE ENGLISH AUCTION

The counterpart to the Dutch auction.The auctioneer starts with a small price. By raising the price in small steps players indicate if they are still willing to pay the new price. It ends when only one person is in the game. He receives the object and pays the price at which the second last bidder dropped out at. *(e.g.: arts in an auction house)*



Further examples

SEALED BID **FIRST** PRICE AUCTION
is a closed auction where the participant with the highest bid receives the good by paying his bid.
(e.g., real estate auction via postal bidding)

SEALED BID **SECOND** PRICE AUCTION
(Vickrey auction) is a closed auction where the participant with the highest bid receives the good by paying the second highest bid.
(e.g., UMTS licences in Australia)

Agents care about utility, not valuation

- Auctions are lotteries, so we must compare expected utility rather than utility.
- *Risk attitude* is described by shape of utility function (see Lecture 3):
  - linear utility function refers to *risk-neutrality*
  - Concave utility function refers to *risk-aversion* $\left(\, u' > 0 \text{ and } u'' < 0 \,\right)$
  - Convex utility function refers to *risk-seeking* $\left(\, u' > 0 \text{ and } u'' > 0 \,\right)$
- The types of utility functions and the associated risk attitudes of agents, are among the most important concepts in Bayesian games, and in particular in auctions.

Most theoretical results about auctions are sensitive to the *risk attitude* of the bidders.

Private value auctions

Basic Auction Environment

- Bidders $i = 1, \dots, n$
- One object to be sold
- Bidder i observes a "signal" $S_i \sim F(\cdot)$, with typical realisation $s_i \in [0, \overline{s}]$
- Bidder's signals $S_i, \dots, S_n$ are independent
- Bidder i's value $v_i(s_i) = s_i$

A set of auction rules will give rise to a game between bidders.

Vickrey (sealed bid second price) auction

Auction Rules:

- Bidders are asked to submit sealed bids $b_1, \cdots, b_n$
- Bidder who submits highest bid wins the object
- Winner pays the amount of the second highest bid

Proposition

In a second price auction, it is a (weakly) dominant strategy to bid one's value: $b_i(s_i) = s_i$

Proof

Bid b_i means i will win $\iff$ the price is below b_i

Bid $b_i > s_i \Rightarrow$ sometimes i will win at price above value

Bid $b_i < s_i \Rightarrow$ sometimes i will loose at price below value

Vickrey auction: Expected revenue

- Seller's revenue equals second highest value.
- Let $S^{i:n}$ denote the ith highest of n draws from distribution F.
- Seller's expected revenue is $\mathbb{E}\left[S^{2:n}\right]$.

Open ascending auction

Auction Rules

- Prices rise continuously from zero
- Bidders have the option to drop out at any point
- Auction ends when only one bidder remains
- Winner pays the ending price

With private values, 'just like' a Vickrey auction!

Sealed bid (first price) auction

Auction Rules

- Bidders submit sealed bids $b_1, \ldots, b_n$
- Bidders who submits the highest bid wins the object
- Winner pays his own bid

Bidders will want to shade bids below their values

Optimal bid for first price auction

Suppose bidders $j \neq i$ bid $b_j = b(s_j)$, $b(\cdot)$ increasing.
Bidder i's expected payoff:

$$U(b_i, s_i) = (s_i - b_i) \cdot Pr\left[b_j = b(S_j) \leq b_i, \forall j \neq i\right]$$

Bidder i chooses b_i to solve:

$$\max_{b_i} (s_i - b_i) F^{n-1}\left(b^{-1}(b_i)\right)$$

where $F(\cdot)$ is the probability that a random draw from F is smaller than $\cdot$.
First order condition (differentiate w.r.t. b_i):

$$0 = (s_i - b_i)(n-1) F^{n-2}\left(b^{-1}(b_i)\right) f\left(b^{-1}(b_i)\right) \frac{1}{b'(b^{-1}(b_i))} - F^{n-1}\left(b^{-1}(b_i)\right)$$

and $f = F'$.

Optimal bid for first price auction

$$0 = (s_i - b_i)(n-1)F^{n-2}\big(b^{-1}(b_i)\big)\, f\big(b^{-1}(b_i)\big)\frac{1}{b'(b^{-1}(b_i))} - F^{n-1}\big(b^{-1}(b_i)\big)$$

At symmetric equilibrium, $b_i = b(s_i)$, first order condition is (dropping subscript i):

$$b'(s) = (s - b(s))(n-1)\frac{f(s)}{F(s)}$$

This differential equation can be solved using the boundary condition $b(0) = 0$:

$$b(s) = s - \frac{\int_0^s F^{n-1}(\tilde{s})d\tilde{s}}{F^{n-1}(s)}$$

Equilibrium in the First Price Auction

- We have found necessary conditions for symmetric equilibrium.
- Verifying that $b(s)$ is an equilibrium is not too hard.
- Also one can show that equilibrium is unique.

Expected First Price Auction Revenue

- Revenue is highest bid $b(s^{1:n})$; expected revenue is $\mathbb{E}\left[b(S^{1:n})\right]$.

$$b(s) = s - \frac{\int_0^s F^{n-1}(\tilde{s})d\tilde{s}}{F^{n-1}(s)} = \frac{1}{F^{n-1}(s)} \int_0^s \tilde{s}F^{n-1}(\tilde{s})d\tilde{s} = \mathbb{E}\left[S^{1:n-1}|S^{1:n-1} \leq s\right]$$

That is, if a bidder has signal s, he sets his bid equal to the expectation of the highest of the other $n-1$ values, conditional on all those values being less than his own.
The expected revenue is:

$$\mathbb{E}\left[b(S^{1:n})\right] = \mathbb{E}\left[S^{1:n-1}|S^{1:n-1} \leq S^{1:n}\right] = \mathbb{E}\left[S^{2:n}\right]$$

First and second price auction yield the same expected revenue!

Revenue equivalence theorem

Theorem (Myerson 1981)

Suppose n bidders have values $s_i, \cdots, s_n$ identically and independently distributed with cdf $F(\cdot)$.
Then any equilibrium of any auction game in which

1. the bidder with the highest value wins the object,
2. a bidder with value 0 gets zero profits,

generates the same revenue in expectation.

Risk neutrality is necessary for revenue equivalence

- Risk-averse agents
  - for bidders:
    Dutch, first-price sealed-bid $\geq$ Vickrey, English
    Compared to a risk neutral bidder, a risk averse bidder will bid higher ("buy" insurance against the possibility of loosing)
    (Utility of winning with a lower bid $<$ utility consequence of loosing the object)
  - For auctioneer:
    Dutch, first-price sealed bid $\leq$ Vickrey, English
- Risk-seeking agents
  - The expected revenue in third-price is greater than the expected revenue in second-price (English)
  - Under constant risk-attitude: $(k+1)$-price is preferable to k- price

Private value is necessary for revenue equivalence

Results for non-private value auctions

- Dutch strategically equivalent to first price sealed bid
- Vickrey not strategically equivalent to English
- All four protocols (Dutch, English, Vickery, first) allocate item efficiently

Theorem: Revenue non-equivalence

With more than 2 bidders, the expected revenues are not the same:
English $\geq$ Vickrey $\geq$ Dutch $=$ First-price sealed bid

A little auction

- We are playing a second price, sealed-bid auction
- You are bidding for a jar of coins with less than 100 CHF
- The winner(s) will play the second highest bid and receive the amount in the jar
- Please all go to the website:

  http://scienceexperiment.online/spa

This was a common value auction!

The winner's curse

- Each bidder must recognize that he/she wins the object only when he/she has the highest signal
- Failure to take into account the bad news about others' signals that come with any victory can lead, on average, to the winner paying more, than the prize is worth
- This is said to happen often in practice

Investor's open secret: "holding is better than trading"

Multi-unit auctions

multiple indistinguishable items for sale

Examples:

- IBM stock
- Barrels of oil
- Pork belies
- Trans-Atlantic backbone bandwidth from NYC to Paris
- …

Multiple-item auctions

multiple distinguishable items for sale

- Auction of multiple, distinguishable items
- Bidders have preferences over item combinations
- Combinatorial auctions
  - Bids can be submitted over item bundles
  - Winner selection: Combinatorial optimization

The revelation principle *(Mechanism Design)*

In a revelation mechanism agents are asked to report their types *(e.g. valuations for the good)*.
Then, an action *(e.g. decision on the winner and his payment)* will be based on the agent's announcement.

In general, agents may cheat about their types, but:

Revelation principle

Any mechanism, that implements certain behavior *(e.g., a good is allocated to the agent with the highest valuation v and he pays* $(1 - \frac{1}{n})v$*)* can be replaced by another mechanism, which implements the same behaviour and where truth-revealing is an equilibrium.

Auction theory – a success story: UK 3G AUCTION

Task: The UK wanted (in 2000) to allocate “air space” for 3G mobile usage

Why an auction is a good choice:

- Utility to companies unknown to government; auction is method most likely allocating resource to those who can use them most valuably (rather than for example a “competition”)
- No room for favoritism and corruption by the government
- If designed appropriately can maximize revenue for auctioneer (i.e., government, tax payer)

Auction theory – a success story: UK 3G AUCTION

The role for auction theorists: Ken Binmore and Paul Klemperer lead a team of researchers to design the auction.

Result:

- 22.5 billion pounds were raised
- Comparable auctions of 3G air space in other west European countries varied from less than 20 dollars per capita in Switzerland to almost 600 dollars per capita in the United Kingdom

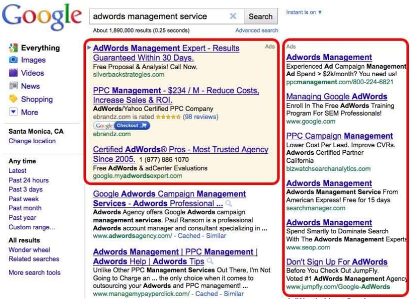
Binmore:  Klemperer: 

Auction theory – a success story: GOOGLE

The role for auction theorists:
Hal Varian leads the economics team designing the auctioning of ad-space.

- Multi-unit
- Multi-item
- Dynamic element (repeated games)
- ...

>50 billion USD revenue per year



Hal Varian on Google auctions:
`https://www.youtube.com/watch?v=PjOHTFRaBWA`

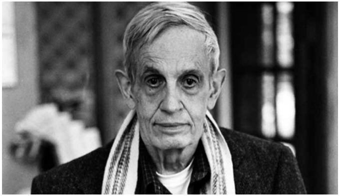
Some introductory texts

- Vijay Krishna: Auction Theory (Academic Press)
- Paul Klemperer: Auction Theory: A guide to the literature (Journal of Economics Survey)
- Tuomas Sandholm COURSE: CS 15-892 Foundations of Electronic Marketplaces (CMU)

What we learned ...

What is game theory?

- A mathematical language to express models of, as Myerson says: “conflict and cooperation between intelligent rational decision-makers”
- In other words, *interactive decision theory* (Aumann)
- Dates back to von Neumann & Morgenstern (1944)
- Most important solution concept: the Nash (1950) equilibrium



The **two** branches of game theory

Non-cooperative game theory

- No binding contracts can be written
- Players are individuals
- Nash equilibrium

Cooperative game theory

- Binding contract can be written
- Players are individuals and coalitions of individuals
- Main solution concepts:
  - Core
  - Shapley value

Shapley value (Shapley 1953)

Axioms. Given some $G(v,N)$, an acceptable allocation/value $x^*(v)$ should satisfy

- **Efficiency.** $\sum_{i\in N} x_i^*(v) = v(N)$
- **Symmetry**. if, for any two players i and j, $v(S\cup i) = v(S\cup j)$ for all S not including i and j, then $x_i^*(v) = x_j^*(v)$
- **Dummy player**. if, for any i, $v(S\cup i) = v(S)$ for all S not including i, then $x_i^*(v) = 0$
- **Additivity**. If u and v are two characteristic functions, then $x^*(v+u) = x^*(v) + x^*(u)$

Von Neumann-Morgenstern

Theorem (von Neumann-Morgenstern)

Let $\succeq$ be a rational (complete & transitive) and continuous preference relation on $X = \Delta(T)$, for any finite set T.
Then $\succeq$ admits a utility function u of the expected-utility form if and only if $\succeq$ meets the axiom of independence of irrelevant alternatives.

Normal form game

Definition: Normal form game

A normal form (or strategic form) game consists of three object:

1. *Players:* $N = \{1, \dots, n\}$, with typical player $i \in N$.
2. *Strategies:* For every player i, a finite set of strategies, S_i, with typical strategy $s_i \in S_i$.
3. *Payoffs:* A function $u_i : (s_1, \dots, s_n) \to \mathbb{R}$ mapping strategy profiles to a payoff for each player i. $u : S \to \mathbb{R}^n$.

Thus a normal form game is represented by the triplet:

$$G = \langle N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$$

Mixed-Strategy Nash Equilibrium

Definition: Mixed-Strategy Nash Equilibrium

A *mixed-strategy Nash equilibrium* is a profile σ^* such that,

$$U_i(\sigma_i^*, \sigma_{-i}^*) \geq U_i(\sigma_i, \sigma_{-i}^*) \text{ for all } \sigma_i \text{ and } i.$$

Nash's equilibrium existence theorem

Theorem (Nash 1951)

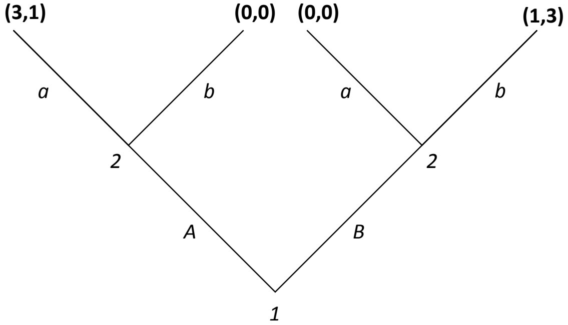
Every finite game has at least one [Nash] equilibrium in mixed strategies.

Dynamic games: Extensive form

Battle of the sexes:

	a	b
A	$3, 1$	$0, 0$
B	$0, 0$	$1, 3$

What if row player (player 1) can decide first?



Game theory and distributed control

Biology:

- Individuals (honeybees)
- Strategies (foraging nectar)
- Outcome (survival)

beecare.bayer.com

CONTROL THEORY:

- Distributed agents (turbines)
- Actions (orientation)
- System performance (energy)

studyindenmark.dk

Experimental game theory

- Aristotle called man a "rational animal" ("zoon logikon" or "zoon logon echon")
- There is a side to human nature which is rational, describable by (corrected) utility maximization
- Utility may include components concerning others' material payoffs too
- There is also a side not describable that way – but instead by heuristics and by learning models
- **Experimental game theory** can shed light on which behavior should be expected in which situations

Firms/workers and their willingness to pay/accept

Firms $i \in F$ and workers $j \in W$ repeatedly look for partners ($|F| = |W| = N$)

- Firm i is willing to pay at most $r_i^+(j) \in \delta\mathbb{N}$ to match worker j
- Worker j is willing to accept at least $r_j^-(i) \in \delta\mathbb{N}$ to match firm i

THANK YOU !