BARGAINING

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Plan for today

1. Bargaining applications
2. Cooperative bargaining solution
3. Noncooperative bargaining program
4. Experimental bargaining

Lecture logic

Topic

- Introduce bargaining
- Illustrations/ applications
- Bridge cooperative and noncooperative game theory (again...)

Appeal

- Bargaining is ubiquitous
- May be useful in real life
- Illustrates the idea of the “Nash program”

Examples of bargaining

Markets:

- Individuals (buyer/seller)
- Strategies (bid/ask certain prices)
- Outcome (profits/losses)

jcrs.com

Splitting:

- Players (partners)
- Strategies (demands)
- Outcome (a split)

mirror.co.uk

Bargaining in real-world markets

Bombay Stock Exchange

Stock market:

- Individuals (buyer/seller)
- Strategies (bid/ask certain prices)
- Outcome (profits/losses)

L'Inde Fantome
(L. Malle 1969)

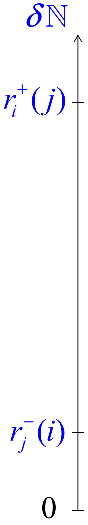
Bargaining over what?

Buyers/sellers and their willingness to pay/accept

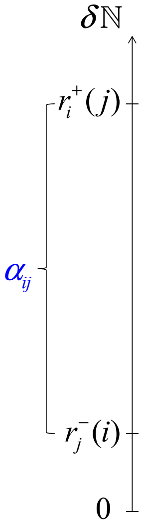
Buyer $\boldsymbol{i} \in \boldsymbol{B}$ and seller $\boldsymbol{j} \in \boldsymbol{S}$ look for partners ($|\boldsymbol{B}| = |\boldsymbol{S}| = \boldsymbol{N}$) – each seller owns exactly one good and each buyer wants exactly one good

- Buyer $\boldsymbol{i}$ is willing to pay at most $\boldsymbol{r_i^+(j)} \in \boldsymbol{\delta}\mathbb{N}$ for the product of seller $\boldsymbol{j}$
- Seller $\boldsymbol{j}$ is willing to accept at least $\boldsymbol{r_j^-(i)} \in \boldsymbol{\delta}\mathbb{N}$ to sell his product to buyer $\boldsymbol{i}$

where $\boldsymbol{\delta} > \boldsymbol{0}$ is the minimum unit ('dollars')



Bargaining over the match value



The match value for the pair (i,j) is

$$\alpha_{ij} = (r_i^+(j) - r_j^-(i))_+$$

Let $\alpha = (\alpha_{ij})_{i\in F, j\in W}$

Normalization.
Let's normalize this value to the 'unit-pie' $\alpha_{ij} = 1$ for some (i,j).

Is there any economic activity more basic than two people dividing a pie?

The pie could symbolize the gains from trade in a market, the surplus generated within a firm, or the benefit from writing a joint paper on economics. Supposing that the nature of the split does not affect the pie's total size, this is a case in which distribution and efficiency is thought not to conflict. Surely, sensible people will come to some agreement rather than backing away from the transaction empty-handed. This argument has permeated economic thinking at least since Edgeworth [1881], and is sometimes referred to as neoclassical bargaining theory (see, e.g., Harsanyi [1987]).

from T. Ellingsen (1997): The Evolution of Bargaining Behavior.

The basic bargaining model

- *Ingredients:*
  - Multiple parties/players
  - A common gain/pie
  - No central authority
  - Bargaining ensues
  - Some outcome is reached

From the analyst's point of view, how do we model this as a "game"?

Two approaches

- *Cooperative:*
  - Multiple parties/players
  - Coalitions form/contract is written
  - Normative axioms are established
  - Outcome is identified
  - Outcome is implemented

- *Noncooperative:*
  - Multiple parties/players
  - Bargaining follows some rules
  - Players act strategically
  - Bargaining takes place
  - Outcome is implemented

Examples

- *Cooperative:*
  - Twins share presents
  - They have identical preferences
  - Twins agree on a splitting rule
  - Sharing fifty-fifty is the only fair rule accepted by both
  - Presents are divided in equal halves
  - Outcome is implemented

- *Noncooperative:*
  - A buyer and a seller meet on the market
  - They have different preferences
  - Buyers make offers
  - Sellers make counteroffers
  - Both try to get the most out of the deal
  - If an offer is accepted, they deal
  - If not, no deal

Compare with our 'cooperative solutions' (Lecture 2)

- *Shapley value:*
  - All players could agree on the axioms
  - They could write an agreement that the SV is implemented
  - Then the outcome would be implemented

- *Core:*
  - When the SV lies inside the core, this seems stable
  - However, as the SV may lie outside the core
  - Or when the core is empty
  - Then there would exist coalitions that perhaps would break the deal

The first formal model (Nash again!)

2-person cooperative bargaining

Nash (1953): Two-Person Cooperative Games. Econometrica 21.

Aside: there were earlier versions due to Edgeworth 1881, Zeuthen 1930 and von Neumann and Morgenstern 1944.

2-person cooperative bargaining

Two person sharing the unit-pie

Basic ingredients:

- players $N = \{1, 2\}$
- outside options $v(i) = o_i \in [0, 1)$ for both $i \in N$
- agreement value $v(N) = 1$

The aim:

- The goal is to reach an agreement (s_1, s_2) such that
- $s_1 + s_2 = 1$ – Pareto efficient
- $s_i \geq o_i$ for all i – Individually rational

Nash bargaining 1

Individual preferences and ***normative postulates***:

- Agents have different preferences $u_i(c)$ s.t.
- $\partial u_i(c)/\partial c > 0$ and
- $\partial^2(u_i(c))/\partial c^2 < 0$
- The outcome that is reached should be "fair"!
- But what is fair?

If everything (including preferences and outside options) is identical, ... easy...
50 : 50 is fair.

Nash bargaining 2

In general, there may be conflict between what is “fair” and what will be reached by strategic bargaining.

Nash program

Derive a framework for noncooperative bargaining, at the end of which the outcome is a *Nash equilibrium* (i.e. such that everyone’s choice is optimal given the choices of others), and that outcome implements a *cooperative solution* concept.

Illustrating the Nash program

- Bargaining sets obtained from a bimatrix game
- Bargaining axioms
- The Nash bargaining solution
- Geometric characterization of the Nash bargaining solution

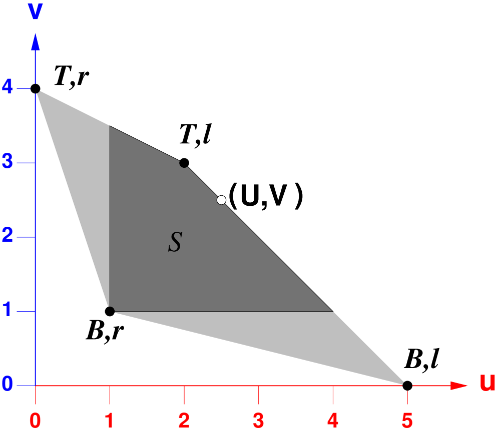
- Splitting a unit pie, concave utility functions
- The ultimatum game
- Alternating offers over several rounds
- Stationary strategies
- The Nash bargaining solution via alternating offers

Toward a cooperative bargaining solution: "The Nash Bargaining Solution"

JF Nash (1950). 'The Bargaining Problem'. ***Econometrica*** $\mathbf{18(2):155-162}$.

Bargaining set from a bimatrix game

I \ II	l	r
T	2, 3	0, $\boxed{4}$
B	$\boxed{5}$, 0	$\boxed{1}$, $\boxed{1}$



Axioms for **Bargaining Set** $S \subset \mathbb{R}^2$

- Threat point $(u_0, v_0) \in S$,
  for all $(u, v) \in S$: $u \geq u_0, v \geq v_0$.
- S is compact (bounded and closed)
- S is convex (via agreed joint lotteries)

Axioms for **Nash Bargaining Solution** $N(S)$

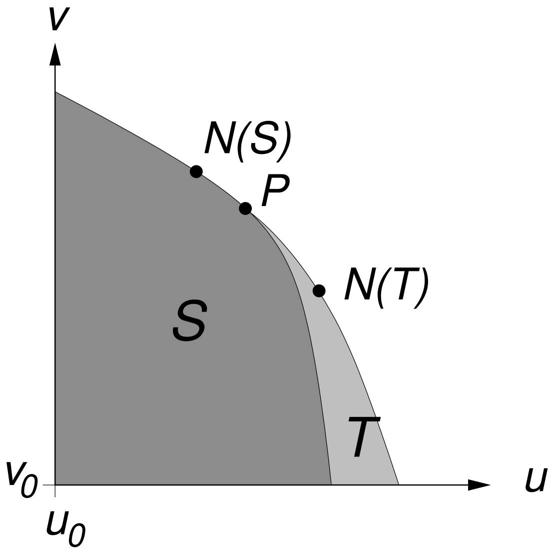
- $N(S) = (U, V) \in S$.
- **Pareto-optimality:** for all $(u, v) \in S$:

  $u \geq U$ and $v \geq V \;\; \Rightarrow \;\; (u, v) = (U, V)$
- **Invariance of utility scaling:** $a, c > 0$,

  $S' = \{(au + b, cv + d) \mid (u, v) \in S\} \;\; \Rightarrow \;\; N(S') = (aU + b, cV + d)$.
- **Symmetry:** if S is symmetric, then so is $N(S)$:

  If $(u, v) \in S$ implies $(v, u) \in S$, then $U = V$.
- **Irrelevance of unused alternatives:** If S, T are bargaining sets with the same threat point and $S \subset T$, then $N(T) \notin S$ or $N(T) = N(S)$.

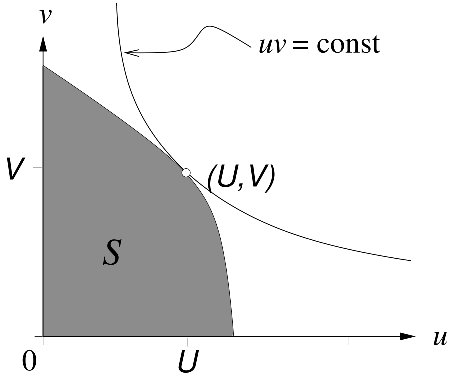
Irrelevance of unused alternatives



The Nash bargaining solution [Nash 1950]

Under the Nash bargaining axioms, every bargaining set $\boldsymbol{S}$ containing a point $(\boldsymbol{u}, \boldsymbol{v})$ with $\boldsymbol{u} > \boldsymbol{u_0}$ and $\boldsymbol{v} > \boldsymbol{v_0}$ has a unique solution $\boldsymbol{N}(\boldsymbol{S}) = (\boldsymbol{U}, \boldsymbol{V})$.

$(\boldsymbol{U}, \boldsymbol{V})$ maximises the following product–
Nash product: $(\boldsymbol{U} - \boldsymbol{u_0})(\boldsymbol{V} - \boldsymbol{v_0})$ for $(\boldsymbol{U}, \boldsymbol{V}) \in \boldsymbol{S}$.



Nash bargaining solution - proof

- **Shift threat point** (u_0, v_0) to $(0, 0)$:

  replace S with $S' = \{(u - u_0, v - v_0) \mid (u, v) \in S\}$

$\Rightarrow$ Nash product maximised as UV (rather than $(U - u_0)(V - v_0)$)

- **re-scale utilities** so that $(U, V) = (1, 1)$:

  replace S with $S' = \{(u/U, v/V) \mid (u, v) \in S\}$.

- **consider** $T = \{(u, v) \mid u \geq 0,\ v \geq 0,\ u + v \leq 2\}$

  $N(T) = (1, 1)$, because T is a symmetric set, and $(1, 1)$ is the only symmetric point on the Pareto-frontier of T.

- Claim: $S \subseteq T \Rightarrow$ (by independence of irrelevant alternatives) $N(S) = N(T)$ because $(1, 1) \in S$.

Proof that $S \subseteq T$

Proof that $S \subseteq T$

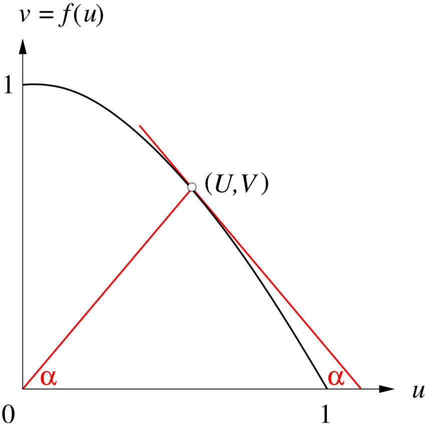
Suppose exists $(\bar{u}, \bar{v}) \in S$, $(\bar{u}, \bar{v}) \notin T \;\; \Rightarrow \;\; \bar{u} + \bar{v} > 2$.

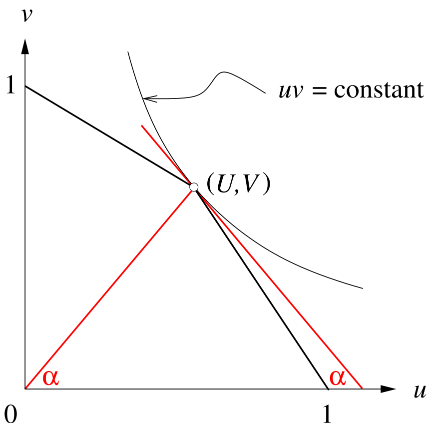
Idea: even if Nash product $\bar{u}\,\bar{v} \leq 1 = UV$, still $\boxed{uv > 1}$ for

$(u, v) = (1 - \varepsilon)(1, 1) + \varepsilon(\bar{u}, \bar{v})$, contracting maximality of UV,

where $(u, v) \in S$ by convexity of S.

Geometric characterization of $\boldsymbol{U}, \boldsymbol{V}$



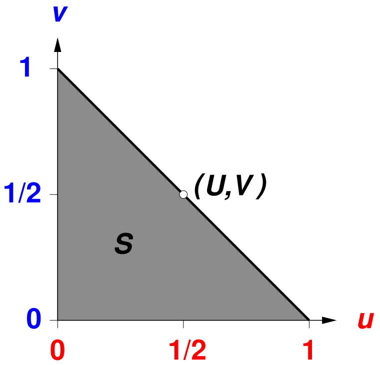


Splitting a unit pie

Suppose player I and player II have to split an amount (a "pie") of one unit into x for player I and y for player II, where

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 1\,.$$

Then this defines in a simple way a bargaining set S if $u = x$ and $v = y$.



Split pie with utility functions

More generally, assume the pie is split into $\boldsymbol{x}$ and $\boldsymbol{y}$ so that

player I receives $\boldsymbol{u}(\boldsymbol{x})$, player II receives $\boldsymbol{v}(\boldsymbol{y})$,

where $\boldsymbol{x} \geq \mathbf{0}$, $\boldsymbol{y} \geq \mathbf{0}$, $\boldsymbol{x} + \boldsymbol{y} \leq \mathbf{1}$. Here

player I has utility function $\boldsymbol{u} : [\mathbf{0}, \mathbf{1}] \to [\mathbf{0}, \mathbf{1}]$

player II has utility function $\boldsymbol{v} : [\mathbf{0}, \mathbf{1}] \to [\mathbf{0}, \mathbf{1}]$

with $\boldsymbol{u}(\mathbf{0}) = \mathbf{0}$, $\boldsymbol{u}(\mathbf{1}) = \mathbf{1}$, $\boldsymbol{v}(\mathbf{0}) = \mathbf{0}$, $\boldsymbol{v}(\mathbf{1}) = \mathbf{1}$,

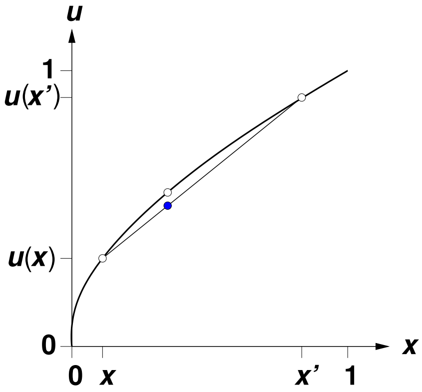
and $\boldsymbol{u}$ and $\boldsymbol{v}$ increasing, continuous. and **concave**.

Concave utility functions

A concave utility function u has "diminishing returns". If u is differentiable this means $u'' \leq 0$, in general

$$(1-p)u(x) + pu(x') \leq u((1-p)x + px')$$

for all x, x' and $p \in [0,1]$.

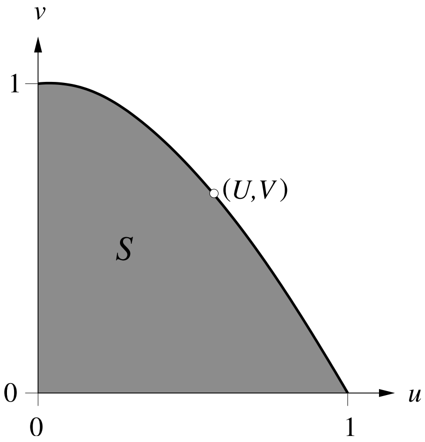


Example
$u(x) = \sqrt{x}$

Convex bargaining set

With concave u and v, the bargaining set S is convex,

$$S = \{(u(x), v(y)) \mid x \geq 0,\ y \geq 0,\ x + y \leq 1\}$$



Example
$u(x) = \sqrt{x}$
$v(y) = y$

Nash bargaining solution

Example $u(x) = \sqrt{x}, \quad v(y) = y$

Pareto-frontier = $\{(u(x), v(1-x) \mid 0 \le x \le 1\}$

The Nash bargaining solution maximizes

$$u(x)v(1-x) = \sqrt{x}(1-x) = x^{1/2} - x^{3/2} \, .$$

Derivative set to zero:

$$0 = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{1/2} = \frac{1}{2}x^{-1/2}(1-3x),$$

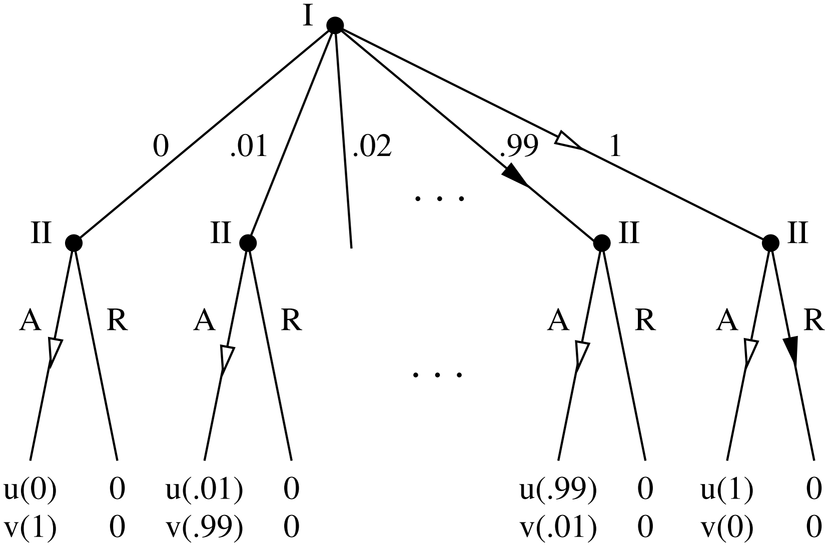
that is, $x = 1/3$ = share for player I, and player II gets $y = 2/3$.

Utilities $(U, V) = (\sqrt{1/3}, 2/3) \approx (0.577, 0.667)$.

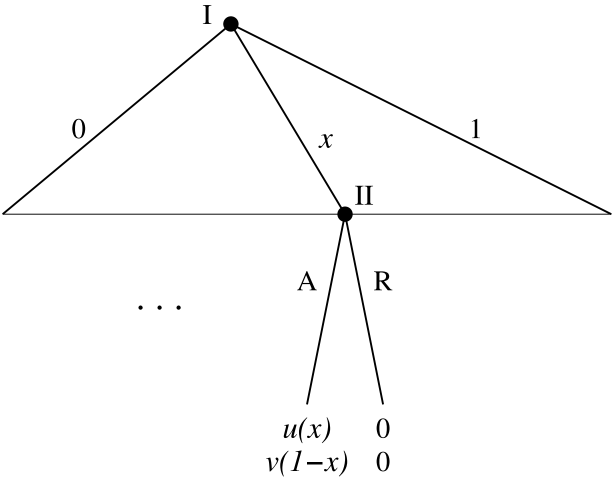
Toward noncooperative foundations: “The Rubinstein Bargaining Model”

A Rubinstein (1982). ‘Perfect Equilibrium in a Bargaining Model’. ***Econometrica*** $\mathbf{50(1):97-109}$.

The ultimatum game



Continuous version of the ultimatum game



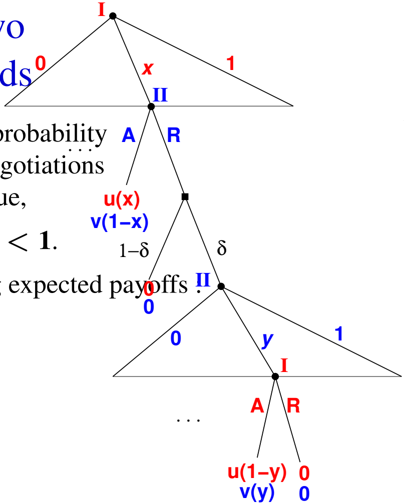
SPNE: player I makes player II **indifferent** between accepting and rejecting, here with $\boldsymbol{x = 1}$, but player II nevertheless accepts.

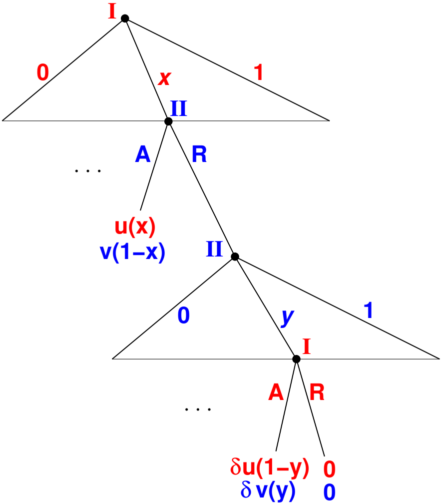
Bargaining in two rounds

$\delta =$ probability that negotiations continue,

$\mathbf{0} < \delta < \mathbf{1}$.

Taking expected payoffs :

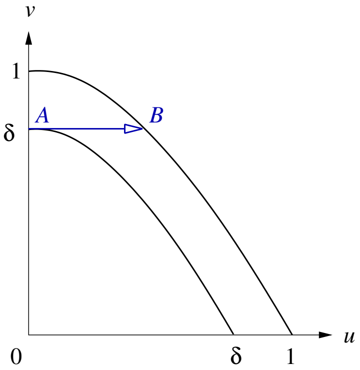




Graphical solution for two rounds

SPNE : in last round, player II makes the ultimatum demand of $y = 1$, player I accepts, player II gets $\delta v(y) = \delta$, player I gets 0.

In previous (first) round, player I makes player II indifferent between accepting and (A) rejecting and making her counterdemand, where she gets δ, by offering $1 - x$ so that (B) $\boxed{v(1 - x) = \delta}$, and player II accepts in round 1, at point B.
Payoffs are $u(x)$, $v(1 - x)$.

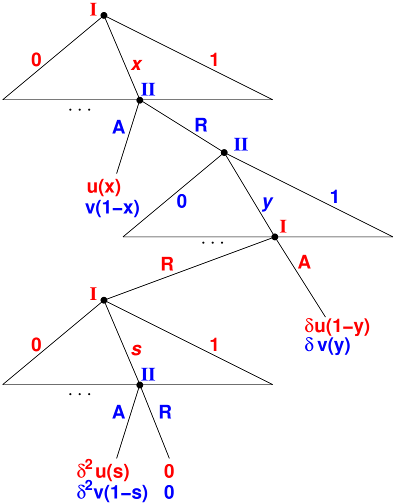


Bargaining in three rounds

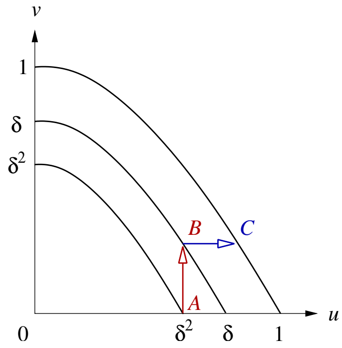
x = demand by player I in round 1

y = counter-demand by player II in round 2

s = counter-counter-demand by player I in last round 3



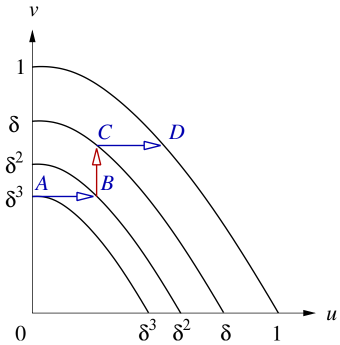
Graphical solution for three rounds



$A \to B: \delta^2 u(1) = \delta u(1-y)$ (round 2, player II chooses y)

$B \to C: \delta v(y) = v(1-x)$ (round 1, player I chooses x)

Graphical solution for four rounds



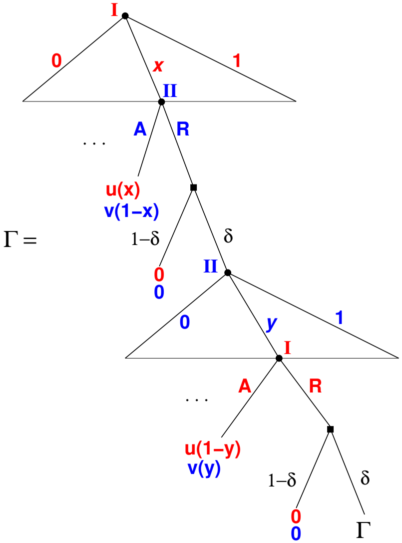
$A \to B: \ \delta^3 v(1) = \delta^2 v(1-s)$ (round 3, player I chooses s)

$B \to C: \ \delta^2 u(s) = \delta u(1-y)$ (round 2, player II chooses y)

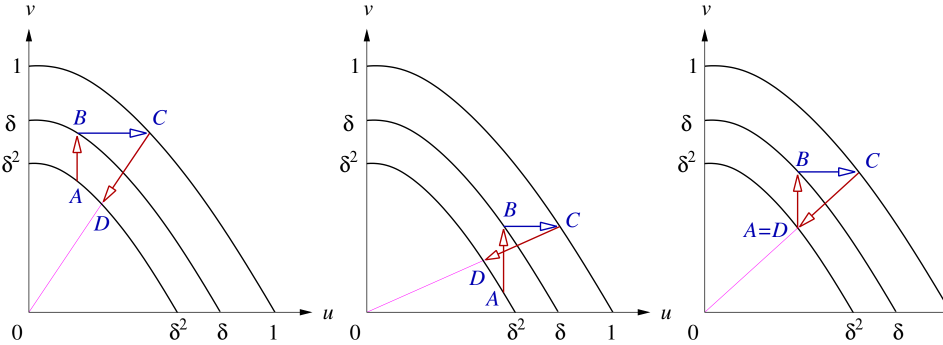
$C \to D: \ \delta v(y) = v(1-x)$ (round 1, player I chooses x)

Infinite number of rounds

look for **stationary** strategies ***x*** and ***y***



Find stationary strategies graphically

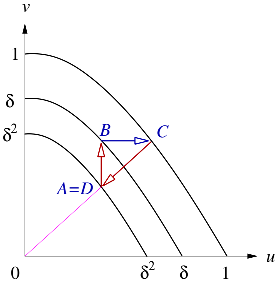


$A \rightarrow B: \delta^2 u(s) = \delta u(1-y)$ (round 2, player II chooses y)

$B \rightarrow C: \delta v(y) = v(1-x)$ (round 1, player I chooses x)

$C \rightarrow D: u(s) = u(x)$? yes! ($\Leftrightarrow$ $s = u$, stationarity)

Characterization of stationary strategies



In rounds 2, 4, 6,...: $\boldsymbol{A} \rightarrow \boldsymbol{B}$: player II demands $\boldsymbol{y}$ so that $\boldsymbol{\delta^2 u(x) = \delta u(1-y)} \Leftrightarrow$ $\boxed{\boldsymbol{\delta u(x) = u(1-y)}}$

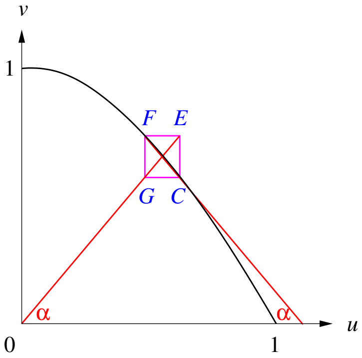
In rounds 1, 3, 5,...: $\boldsymbol{B} \rightarrow \boldsymbol{C}$: player I demands $\boldsymbol{x}$ so that $\boxed{\boldsymbol{\delta v(y) = v(1-x)}}$
(two equations with two unknowns)

The Nash bargaining solution via alternating offers

Theorem

As $\delta \to 1$, the payoffs $u(x), v(y)$ for the stationary strategies x and y of alternating offers with an infinite number of rounds tend to the **Nash bargaining solution** U, V that maximizes UV for $U = u(x)$, $V = v(1 - x)$.

Graphical proof



$$
\begin{aligned}
C &= (u(x), v(1-x)),\\
F &= (u(1-y), v(y)),\\
E &= (u(x), v(y)),\\
G &= (\delta u(x), \delta v(y)).
\end{aligned}
$$

$G \to C:$

$$\boxed{\delta v(y) = v(1-x)},$$

$G \to F:$

$$\boxed{\delta u(x) = u(1-y)}$$

$\Rightarrow$ ***CEFG*** is a **rectangle** with diagonals ***FC*** and ***GE*** of equal slope α.

Bargaining evidence from laboratory experiments

AE Roth (1995). 'Bargaining Experiments.' In Handbook of Experimental Economics, edited by John Kagel and Alvin E. Roth, 253-348. Princeton University Press.

VL Smith (1962). 'An Experimental Study of Competitive Market Behavior.' Journal of Political Economy 70(2): 111-137.

Ultimatum Game Bargaining

- recall last lecture

As in the Rubinstein bargaining model (with only one bargaining round)

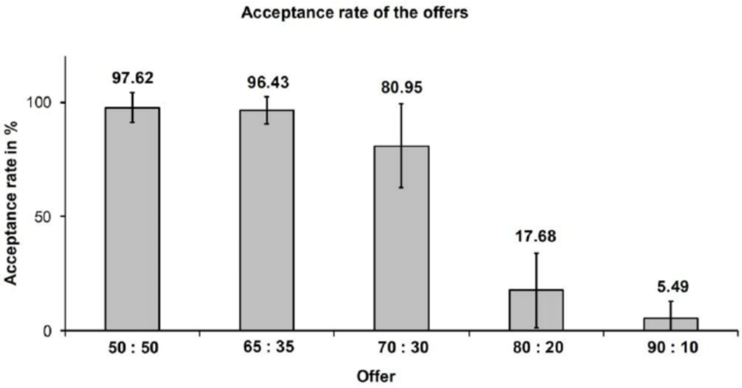
1. the proposer (player 1) suggests a split between him and the receiver (player 2)
2. Player 2 can either accept or reject:
    1. If he accepts, the shares proposed by player 1 realize
    2. If he rejects, both players receive nothing.

- **Nash equilibria:** any split supportable as a Nash equilibrium
- Unique **subgame-perfect Nash equilibrium** prediction: (1 all, 2 nothing)

Recap: features and evidence

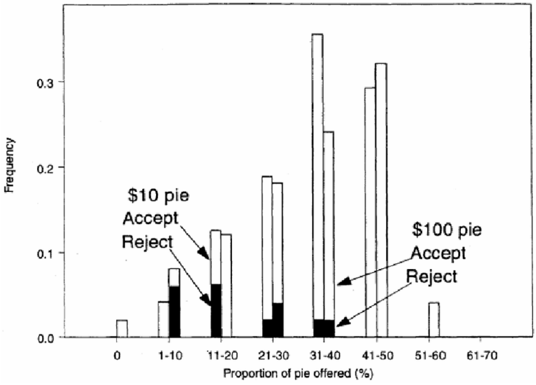
- Rejection by the responder kills own and other's payoff
- Any positive proposal, expecting acceptance, seems like a 'gift';
- however, expecting (off the SPNE-path) rejection if one's offer is too low, a substantial proposal may be strategically rational
- hence, for the responder, it may be rational to have a **rejection reputation**
- Meta-analysis suggests
  - proposals of roughly 40%;
  - high rejection rates for proposals under 20%, intermediate rejection rates for proposals of 20%-40%, and almost zero rejection rates for proposals >40%
  - rates vary with stakes, matching protocol, etc.

Recap 1: acceptance rates



from Hollmann et al., PLoS ONE 2011

Recap 2: offers



from Hoffman et al., IJGT 1996

For one last time, let's play a large single-item economy

1. **Players:** All of you.
2. **Rules of the game:** See instructions.
3. Two individuals (who end up trading with each other) will be paid their payoffs in CHF.

https://scienceexperiment.online/vernon/vote or scan

Thanks!

As always, please contact me under hnax@ethz.ch if you have questions.