INTRODUCTION TO GAME THEORY

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A game

Rules:

1. **Players:** All of you: **https://scienceexperiment.online/beautygame/vote**
2. **Actions:** Choose a number between 0 and 100
3. **Outcome:** The player with the number closest to half the average of all submitted numbers wins.
4. **Payoffs:** He will receive his number in CHF, which I will pay out right after the game.
5. In case of several winners, divide payment by number of winners and pay all winners.

Structure of today's lecture

- Part 1: A "sort-of" introduction to the theory of games
- Part 2: Course admin:
  - Aims and requirements
  - Talk schedule

Acknowledgments

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- Dirk Helbing (ETHZ)

Game theory

A tour of its people, applications and concepts

1. von Neumann
2. Nash
3. Aumann, Schelling, Selten, Shapley
4. Today

John von Neumann (1903-1957)

What is game theory?

- A mathematical language to express models of, as Myerson says: "conflict and cooperation between intelligent rational decision-makers"
- In other words, *interactive decision theory* (Aumann)
- Dates back to von Neumann & Morgenstern (1944)
- Most important solution concept: the Nash (1950) equilibrium

Games and Non-Games

What is a game? And what is not a game?

Uses of game theory

- *Prescriptive* agenda versus *descriptive* agenda
- “Reverse game theory”/mechanism design:
    - “in a design problem, the goal function is the main given, while the mechanism is the unknown.” (Hurwicz)
- The mechanism designer is a game designer. He studies
    - What agents would do in various games
    - And what game leads to the outcomes that are most desirable

Game theory revolutionized several disciplines

- Biology (evolution, conflict, etc.)
- Social sciences (economics, sociology, political science, etc.)
- Computer science (algorithms, control, etc.)

- game theory is now applied widely (e.g. regulation, online auctions, distributed control, medical research, etc.)

Its impact in economics (evaluated by Nobel prizes)

- 1972: Ken Arrow – general equilibrium
- 1994: John Nash, Reinhard Selten, John Harsanyi – solution concepts
- 2005: Tom Schelling and Robert Aumann – evolutionary game theory and common knowledge
- 2007: Leonid Hurwicz, Eric Maskin, Roger Myerson – mechanism design
- 2009: Lin Ostrom – economic governance, the commons
- 2012: Al Roth and Lloyd Shapley – market design
- 2014: Jean Tirole – markets and regulation
- 2016: Oliver Hart and Bengt Holmström – contract theory

Part 1: game theory

"Introduction" / Tour of game theory

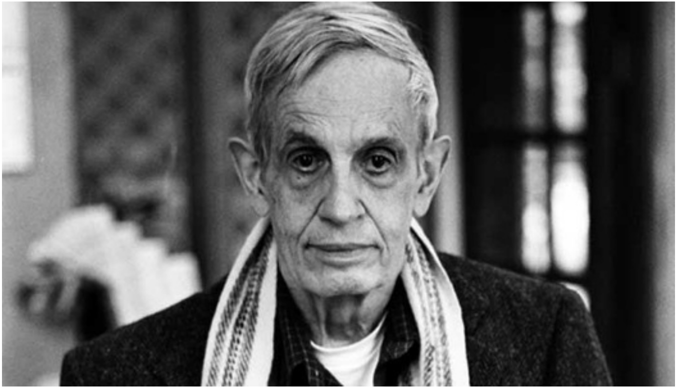
Non-cooperative game theory

- No binding contracts can be written
- Players are individuals
- Main solution concepts:
  - Nash equilibrium
  - Strong equilibrium

Cooperative game theory

- Binding contract can be written
- Players are individuals and coalitions of individuals
- Main solution concepts:
  - Core
  - Shapley value

Noncooperative game theory



John Nash (1928-2015)

A noncooperative game (normal-form)

- **players**: $N = \{1, 2, \ldots, n\}$ (finite)
- **actions/strategies**: (each player chooses s_i from his own finite strategy set; S_i for each $i \in N$)
  - resulting strategy combination: $s = (s_1, \ldots, s_n) \in (S_i)_{i \in N}$
- **payoffs**: $u_i = u_i(s)$
  - payoffs resulting from the outcome of the game determined by s

Some 2-player examples

- Prisoner's dilemma – social dilemma, tragedy of the commons, free-riding
  - Conflict between individual and collective incentives
- Harmony – aligned incentives
  - No conflict between individual and collective incentives
- Battle of the Sexes – coordination
  - Conflict and alignment of individual and collective incentives
- Hawk dove/Snowdrift – anti-coordination
  - Conflict and alignment of individual and collective incentives
- Matching pennies – zero-sum, rock-paper-scissor
  - Conflict of individual incentives

		Player 2	
		Heads	Tails
Player 1	Heads	1,-1	-1,1
	Tails	-1,1	1,-1

Matching pennies

	Confess A	Stay quiet A
Confess B	A: -6, B: -6	A: -10, B: 0
Stay quiet B	A: 0, B: -10	A: -2, B: -2

Prisoner's dilemma

		WOMAN	
		Boxing	Shopping
MAN	Boxing	2,1	0,0
	Shopping	0,0	1,2

Battle of the sexes

		Player 2	
		Hawk	Dove
Player 1	Hawk	-2,-2	4,0
	Dove	0,4	2,2

Hawk-Dove game

		Company B	
		Cooperate	Not Cooperate
Company A	Cooperate	9,9	4,7
	Not Cooperate	7,4	3,3

Harmony game

Equilibrium

Equilibrium/solution concept:

An **equilibrium/solution** is a rule that maps the structure of a game into an equilibrium set of strategies s^*.

Nash Equilibrium

Definition: Best-response

Player i's **best-response** (or, reply) to the strategies s_{-i} played by all others is the strategy $s_i^* \in S_i$ such that

$$u_i(s_i^*, s_{-i}) \geqslant u_i(s_i', s_{-i}) \quad \forall s_i' \in S_i \text{ and } s_i' \neq s_i^*$$

Definition: (Pure-strategy) Nash equilibrium

All strategies are *mutual best responses*:

$$u_i(s_i^*, s_{-i}) \geqslant u_i(s_i', s_{-i}) \quad \forall s_i' \in S_i \quad and \quad s_i' \neq s_i^*$$

		Confess A	Stay quiet A
Confess	B	A: -6, B: -6	A: -10, B: 0
Stay quiet	B	A: 0, B: -10	A: -2, B: -2

Prisoner's dilemma: both players confess (defect)

		WOMAN	
		Boxing	Shopping
MAN	Boxing	2,1	0,0
	Shopping	0,0	1,2

Battle of the sexes: coordinate on either option

		Player 2	
		Heads	Tails
Player 1	Heads	1,-1	-1,1
	Tails	-1,1	1,-1

Matching pennies: none (in pure strategies)

		Player 2	
		Hawk	Dove
Player 1	Hawk	-2,-2	4,0
	Dove	0,4	2,2

Hawk-dove: either of the two hawk-dove outcomes

		Company B	
		Cooperate	Not Cooperate
Company A	Cooperate	9,9	4,7
	Not Cooperate	7,4	3,3

Harmony: both cooperate

Pure-strategy N.E. for our 2-player examples

- Prisoner's dilemma – social dilemma
  - Unique NE – socially undesirable outcome
- Harmony – aligned incentives
  - Unique NE – socially desirable outcome
- Battle of the Sexes – coordination
  - Two NE – both Pareto-optimal
- Hawk dove/Snowdrift – anti-coordination
  - Two NE – Pareto-optimal, but perhaps Dove-Dove "better"
- Matching pennies – zero-sum, rock-paper-scissor
  - No (pure-strategy) NE

How about our initial game

Remember the rules were:

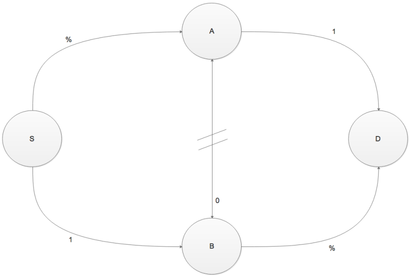
1. Choose a number between 0 and 100
2. The player with the number closest to half the average of all submitted numbers wins his number in CHF

What is the Nash Equilibrium?

0

Actually, there are some Nash equilibria where players play 1 due to the fact that you earn 0 when you play 0... we will get back to this in detail later.

Braess' Paradox



New road worsens congestion!

The story:

- 60 people travel from S to D
- Initially, there is no middle road. The NE is such that 30 people travel one way, the others the other way, and each driver drives 90 mins.
- A middle road is build. This road is super efficient. Now everyone will use it and drive the same route, and the NE will worsen to 119/120 mins.

Cooperative games

The Nash equilibrium may not coincide with the outcome that is collectively preferable. Can players “cooperate" to achieve such an outcome?

- Suppose players can write **binding agreements** and directly **transfer utility**– e.g.:
  - *Contract 1:* Player 1 plays ‘Hawk’, player 2 plays ‘Dove’. Of the total payoffs, 1 and 2 receive equal shares.

  or

  - *Contract 2:* Both players play ‘Boxing’. Of the total payoffs, Man gets 1.6 and Woman gets 1.4.

Then the **value** of the game in terms of a cooperative agreement is generally greater than the sum of the payoffs from the Nash equilibrium.

	Confess A	Stay quiet A
Confess B	-6, -6	0, -10
Stay quiet B	-10, 0	-2, -2

$v(12) = -2 - 2 = -4$
$v(1) = v(2) = -6$
Cooperative value=$v(12) > v(1) + v(2)$ =Nash equilibrium payoffs

		WOMAN	
		Boxing	Shopping
MAN	Boxing	2,1	0,0
	Shopping	0,0	1,2

$v(12) = 1 + 2 = 3$
$v(1) = v(2) = 0$
Cooperative value=Nash equilibrium payoffs=$v(12) > v(1) + v(2)$: payoffs can be split differently/more evenly

		Dawn	
		Hawk	Dove
Gary	Hawk	-2,-2	4,0
	Dove	0,4	2,2

$v(12) = 4 + 0 = 2 + 2 = 4$
$v(1) = v(2) = -2$
Cooperative value=Nash equilibrium payoffs=$v(12) > v(1) + v(2)$: payoffs can be split differently/more evenly, achievable by dove-dove

		Company B	
		Cooperate	Not Cooperate
Company A	Cooperate	9,9	4,7
	Not Cooperate	7,4	3,3

$v(12) = 9 + 9 = 18$
$v(1) = v(2) = 3$
Cooperative value=Nash equilibrium payoffs=$v(12) > v(1) + v(2)$, but payoffs can be split differently/more evenly

Part 2: course admin

- Information about the course, and materials/slides of speakers, will be made available at
  http://www.coss.ethz.ch/education/GT.html
- Also, please contact us directly if you have any questions about the course:
  - Heinrich: hnax@ethz.ch
  - Bary: bpradelski@ethz.ch

Drop-In Office Hours

Heinrich Nax
Monday
20.02.; 27.02.; 06.03; 03.04.; 10.04.
15:00-16:00
CLD C3

Bary Pradelski
Monday
13.03.; 20.03.; 27.03; 08.05.; 15.05.
10:00-11:00
CLD C5

Schedule (preliminary) I

1) 20.02.	Introduction: a quick tour of game theory	Heinrich Nax
2) 27.02.	Cooperative game theory •Core and Shapley value •Matching markets	Heinrich Nax
3) 06.03.	Non-cooperative game theory: Normal form •Utilities •Best replies	Bary Pradelski
4) 13.03.	The Nash equilibrium •Proof •Interpretations and refinements	Bary Pradelski
5) 20.03.	Non-cooperative game theory: dynamics •Sub-game perfection and Bayes-Nash equilibrium •Repeated games	Bary Pradelski
	PROBLEM SET 1	
6) 27.03.	Game theory: evolution •Evolutionary game theory •Algorithms in computer science (Price of anarchy)	Bary Pradelski

Schedule (preliminary) II

7) 03.04.	Experimental game theory •Observing human behavior/experiments •Behavioral game theory	Heinrich Nax
8) 10.04.	Applications •Common pool resources •Distributed control	Heinrich Nax
9) 08.05.	Bargaining •Solution concepts •Nash program	Heinrich Nax
10) 15.05.	Auctions •English, Dutch, Sealed, Open •Equivalence and Real-world examples: 3G, Google, etc	Bary Pradelski
11) 22.05.	EXAM	
12) 29.05.	The diffusion of social and technological innovations	H. Peyton Young

Requirements

1. Regularly attend and, please!, participate in seminar
2. Problem set:
    1. does not count toward final mark
3. **Exam: 22/5/2017**

Finally, let's play again!

You remember the game:

1. Choose a number between 0 and 100
   - Submit this number again: **https://scienceexperiment.online/beautygame/vote**
2. The player with the number that is closest to half the average of all submitted numbers wins his number in CHF (divided by number of winners)

Nash equilibrium:

As a **non-cooperative game**, …the Nash Equilibrium has total earnings of 0 (or 1).

How about our initial game as a cooperative game

Cooperate or not?

- If all players submit 0, the average is 0: 0 earnings
- If all players submit 100, the average is 100: each player earns 100/n
- Cooperatively, total earning could be 100!
- But what if all others submit 100, but one guys submits 99?
  - Then he wins and his earnings will be 99 instead of 100/n …

Cooperative values:
$v(N) = 100$
$v(i) = 0$

THANKS EVERYBODY
See you next week!
and keep checking the website for new materials as we progress:
http://www.coss.ethz.ch/education/GT.html